THE MOTION OF GASES IN THE SUN'S ATMOSPHERE

PART I. ON THE MECHANISM OF FORMATION OF SOLAR DARK MARKINGS

By A. K. DAS

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ABSTRACT. In this paper the dynamics of a mass of gas ejected from the body of the sun has been studied and the equations of motion of a material particle in a system of rotating coordinates with origin on the surface of the sun have been derived for two cases (1) when the particle is ejected from the photospheric or any other level where the angular velocity of rotation is the same as that observed on the sun's surface and (2) when the particle is ejected from a level where the angular velocity is higher than the angular velocity at the surface. The set of equations for case (2) have been shown to be applicable to a mass of gas ejected from the interior of the sun, that is from levels below the photosphere. Since prominences are formed, according to the growing view held at present by many solar physicists, by the gases ejected from the sub-photospheric levels, these equations should explain at least the broad features of prominences and dark markings. It has been shown in this paper that many of the hitherto ill-explained or un-explained features of dark markings, in particular, can be quantitatively explained on the basis of the above equations. Indications have also been given how these equations should be equally useful in the understanding of several other solar phenomena not dealt with in this paper.

The phenomena revealed by spectroheliograms and photoheliograms are so varied that up till now it has not been possible to construct a unified theory which can account perfectly satisfactorily for all the peculiarities observed on the solar disc. Even the most striking feature of the monochromatic disc photographs, namely, the granulations does not seem to be thoroughly understood, although it is evident from the works of Deslandres, Unsold and others that these granulations which give rise to the "photospheric network" must be the result of convection. In the present state of our knowledge the only way to progress appears therefore to be to restrict one's attention to some of the most salient features and to try to evolve a simple hypothesis which can yield these observed properties through straightforward deduction. In this paper attention is confined to some features of the H_α dark markings and a mechanism is put forward to explain these hitherto ill-explained, and in some cases unexplained, features. The proposed mechanism appears to be quite general and helps the understanding of certain other solar phenomena* which are not very

* These will be considered in the later papers of this series.

easily understandable on the basis of existing solar theories. The following features of the dark markings are considered in the present paper :

(*a*) the predominantly linear appearance of the dark markings, which led Deslandres to call them " filaments " ;

(*b*) the tendency of the direction of the dark markings to change from a direction along a meridian in equatorial regions by steadily inclining more and more towards the east until in latitudes higher than about $35°$ the markings lie nearly along a parallel of latitude ;

(*c*) the tendency of high latitude dark markings to have their directions nearly parallel to the parallels of latitude even at their first appearance ;

(*d*) the predominantly westward tilt of the tops of dark markings of the low and middle latitudes, the average inclination of a dark marking to the vertical being of the order of $10°$;

(*e*) the tendency of dark markings to be short in length in the equatorial regions.

It is well established that dark markings are caused by the absorption of photospheric light by the masses of gas which exist as prominences over the solar disc. The appearance of any dark marking is therefore determined by the way in which the absorbing gas is distributed in the prominence concerned and consequently in order to understand the features of the dark markings enumerated in the foregoing paragraph we must form a correct picture of the mode of formation of a prominence. For this purpose an examination of disc photographs is not very helpful although in some cases the dark markings, particularly those of very high latitudes, indicate how the gases responsible for the absorption of photospheric light have emerged from the photosphere. But even a casual examination of limb photographs cannot fail to show that the mass of gas forming a prominence has been ejected from a limited area of the disc. It is therefore natural to assume that a stable prominence is usually formed by a jet of gas issuing from a very small disturbed area of the photosphere or even from below the photosphere. If this hypothesis regarding the mode of formation of a prominence be true, we should be able to understand a good deal of the peculiarities of prominences and consequently of dark markings by considering the dynamics of a jet of gas ejected from the body of the sun. A complete solution of this dynamical problem is highly complicated, but fortunately we can draw several useful conclusions from quite elementary considerations. The problem is strictly one of hydrodynamics, but simple considerations of particle dynamics appear to be sufficient for the purposes of preliminary investigation.

Our problem in its simplified form is the consideration of the motion of a particle ejected from the body of the sun which we regard as a rigid sphere rotating about its polar axis. The equations of motion of a material particle of

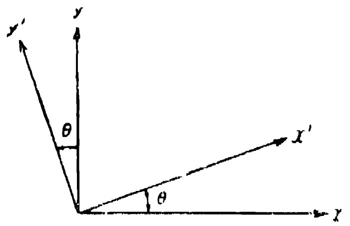

FIGURE 1

mass m in a system of stationary, rectangular coordinates x, y, z are

$$X = m.\frac{du}{dt} = m.\frac{d^2x}{dt^2},$$

$$Y = m.\frac{dv}{dt} = m.\frac{d^2y}{dt^2}, \quad \ldots \quad (1)$$

$$Z = m.\frac{dw}{dt} = m.\frac{d^2z}{dt^2},$$

where X, Y, Z, are components of the force causing the motion of the particle and u, v, w, are the components of the velocity in the directions x, y, z, respectively. But the gases on the sun move with respect to a rotating system of coordinates, and therefore in order to approximate to solar conditions we have to consider the motion of our particle in a coordinate system x', y', z', rotating with a constant angular velocity ω equal to the angular velocity of the sun as observed on its surface. Let the origin of our rotating system coincide with that of the stationary system of coordinates and let the axis of rotation z' coincide with the z-axis of the stationary system. We note that the rotation of the sun is counterclockwise if we look towards the equator from the north pole of the sun ; ω is therefore positive.

Now if θ is the angle between the x-axis and the x'-axis then we have (see Fig. 1)

$$\left.\begin{array}{lll} \alpha_1 = \cos\theta, & \beta_1 = \sin\theta, & \gamma_1 = 0 \\ \alpha_2 = -\sin\theta, & \beta_2 = \cos\theta, & \gamma_2 = 0 \\ \alpha_3 = 0, & \beta_3 = 0, & \gamma_3 = 1 \end{array}\right\} \quad \ldots \quad (2)$$

where α_1, β_1, γ_1, α_2, β_2, γ_2, α_3, β_3, γ_3 are the direction-cosines of the three rotating axes with respect to the stationary axes. We know that according to the general rules of transformation from a stationary system of coordinates x, y, z to a system of coordinates in relative motion x', y', z', the following relations must hold :

$$\left.\begin{array}{l} x' = \alpha_1(x - x_0) + \beta_1(y - y_0) + \gamma_1(z - z_0) \\ y' = \alpha_2(x - x_0) + \beta_2(y - y_0) + \gamma_2(z - z_0) \\ z' = \alpha_3(x - x_0) + \beta_3(y - y_0) + \gamma_3(z - z_0) \end{array}\right\} \quad \ldots \quad (3)$$

and

$$X' = \alpha_1 X + \beta_1 Y + \gamma_1 Z$$
$$Y' = \alpha_2 X + \beta_2 Y + \gamma_2 Z \qquad \ldots \quad (4)$$
$$Z' = \alpha_3 X + \beta_3 Y + \gamma_3 Z$$

where x_0, y_0, z_0 are the coordinates of the origin of the primed system and X, Y, Z and X′, Y′, Z′ are the components of the force in the two systems respectively. In the case under consideration $x_0 = 0$, $y_0 = 0$, $z_0 = 0$; therefore we have from (2) and (3)

$$\left.\begin{aligned} x' &= x \cos\theta + y \sin\theta \\ y' &= -x \sin\theta + y \cos\theta \\ z' &= z. \end{aligned}\right\} \qquad (5)$$

Putting $\theta = \omega t$ and differentiating with respect to t we have from (5)

$$\left.\begin{aligned} u' &= u \cos\theta - x \sin\theta . \frac{d\theta}{dt} + v \sin\theta + y \cos\theta . \frac{d\theta}{dt} \\ &= u \cos\theta + v \sin\theta + (y \cos\theta - x \sin\theta) . \frac{d\theta}{dt} \\ v' &= -u \sin\theta - x \cos\theta . \frac{d\theta}{dt} + v \cos\theta - y \sin\theta . \frac{d\theta}{dt} \\ &= -u \sin\theta + v \cos\theta - (x \cos\theta + y \sin\theta) . \frac{d\theta}{dt} \\ w' &= w, \end{aligned}\right\} \qquad (6)$$

whence by using (5) we get

$$\left.\begin{aligned} u' &= u \cos\theta + v \sin\theta + \omega y' \\ v' &= -u \sin\theta + v \cos\theta - \omega x' \\ w' &= w. \end{aligned}\right\} \qquad (7)$$

Differentiating (7) with respect to t we have

$$\frac{du'}{dt} = \frac{du}{dt} . \cos\theta - u \sin\theta . \frac{d\theta}{dt} + \frac{dv}{dt} . \sin\theta + v \cos\theta . \frac{d\theta}{dt} + \omega v'$$

$$= \frac{du}{dt} \cos\theta + \frac{dv}{dt} . \sin\theta + \omega(v' - u \sin\theta + v \cos\theta)$$

$$\frac{dv'}{dt} = -\frac{du}{dt} . \sin\theta - u \cos\theta . \frac{d\theta}{dt} + \frac{dv}{dt} . \cos\theta - v \sin\theta . \frac{d\theta}{dt} - \omega u'$$

$$= -\frac{du}{dt} \sin\theta + \frac{dv}{dt} \cos\theta - \omega(u' + u \cos\theta + v \sin\theta)$$

$$\frac{dw'}{dt} = \frac{dw}{dt} ,$$

whence using (7) we have

$$\begin{aligned} \frac{du'}{dt} &= \frac{du}{dt}.\cos\theta + \frac{dv}{dt}.\sin\theta + 2\omega v' + \omega^2 x' \\ \frac{dv'}{dt} &= -\frac{du}{dt}.\sin\theta + \frac{dv}{dt}.\cos\theta - 2\omega u' + \omega^2 y' \\ \frac{dw'}{dt} &= \frac{dw}{dt} \end{aligned} \tag{8}$$

Multiplying equations (8) by m we obtain by virtue of the relations (4) and (1) and by ignoring the primes since only primed quantities occur

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} &= X + 2m\omega v + m\omega^2 x \\ m\frac{d^2y}{dt^2} &= Y - 2m\omega u + m\omega^2 y \\ m\frac{d^2z}{dt^2} &= Z\,. \end{aligned}\right\} \tag{9}$$

These are the equations of motion of a material particle in a rotating system of coordinates, but we notice that the equations (9) refer to the motion of a particle on a rotating sphere the centre of which coincides with the origin of the system of coordinates. In order to make the equations comparable with observation we have to transform them to a rotating system of coordinates whose origin is at some point on the surface of the rotating sphere.

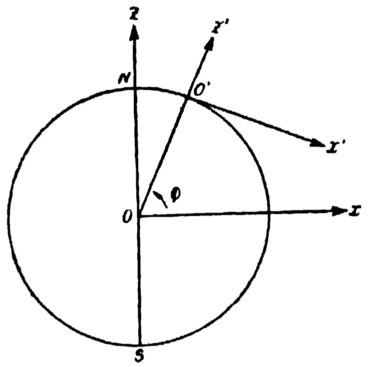

FIGURE 2

Let o' be a point on the surface of the sphere which we choose as the origin of our system of rotating co-ordinates. For simplicity let o' be on the $x-z$ plane which is the plane of Fig. 2, and let the z'-axis point radially outwards, the y'-axis parallel to the y-axis, *i.e.*, towards the back of the paper, so that the x'-axis lies in the plane of the paper. Now, since the z-axis coincides with the axis of rotation, the x-axis represents the equator. Then in the co-ordinate system with origin at o' the positive z'-axis points vertically upwards, the positive y'-axis is towards the west * and the positive x'-axis points towards the south. Also the angle ϕ between the equator and the radius oo' represents the heliographic latitude, positive for the northern and negative for the southern hemisphere of the sun. The coordinates of o' in the previous system are

$$x_0 = R\cos\phi, \quad y_0 = 0, \quad z_0 = R\sin\phi,$$

where R is the radius of the rotating sphere (sun). Also the direction cosines of the primed axes (*i.e.*, of the system with origin at o') with respect to the unprimed axes (*i.e.*, of the system with origin at o) are

$$\begin{aligned} \alpha_1 &= \sin\phi, & \beta_1 &= 0, & \gamma_1 &= -\cos\phi, \\ \alpha_2 &= 0, & \beta_2 &= 1, & \gamma_2 &= 0, \\ \alpha_3 &= \cos\phi, & \beta_3 &= 0, & \gamma_3 &= \sin\phi. \end{aligned}$$

Now, since the new system is rigidly fixed to the former system of co-ordinates, *i.e.*, x_0, y_0, z_0 and the nine direction cosines are independent of the time t, the following relation must hold

$$\frac{du'}{dt} = \alpha_1 \frac{du}{dt} + \beta_1 \frac{dv}{dt} + \gamma_1 \frac{dw}{dt}.$$

Therefore we have

$$\frac{du'}{dt} = \frac{du}{dt}\sin\phi - \frac{dw}{dt}\cos\phi,$$

$$\frac{dv'}{dt} = \frac{dv}{dt}$$

$$\frac{dw'}{dt} = \frac{du}{dt}\cos\phi + \frac{dw}{dt}\sin\phi.$$

Multiplying these equations by m and using the values of $\frac{du}{dt}$, $\frac{dv}{dt}$, $\frac{dw}{dt}$ from the equations (9) and by expressing all unprimed quantities in terms of the primed quantities with the help of the general relations

$$\begin{aligned} x - x_0 &= \alpha_1 x' + \alpha_2 y' + \alpha_3 z' \\ y - y_0 &= \beta_1 x' + \beta_2 y' + \beta_3 z' \\ z - z_0 &= \gamma_1 x' + \gamma_2 y' + \gamma_3 z' \end{aligned}$$

* Our coordinate system is right-handed and when we say east or west we always refer to geographical east and west. But when we say 'southward' we refer to the south pole of the sun, that is to say, in northern hemisphere 'southward' means a direction along a meridian from the north pole to the equator of the sun.

$$u = \alpha_1 u' + \alpha_2 v' + \alpha_3 w'$$
$$X = \alpha_1 X' + \alpha_2 Y' + \alpha_3 Z'$$
$$Y = \beta_1 X' + \beta_2 Y' + \beta_3 Z'$$
$$Z = \gamma_1 X' + \gamma_2 Y' + \gamma_3 Z'$$

which must hold between the two systems of coordinates we have

$$\begin{aligned}
x &= R\cos\phi + x'\sin\phi + z'\cos\phi \\
y &= y' \\
u &= u'\sin\phi + w'\cos\phi \\
v &= v' \\
X &= X'\sin\phi + Z'\cos\phi \\
Y &= Y' \\
Z &= -X'\cos\phi + Z'\sin\phi\ .
\end{aligned}$$

Now since only primed quantities occur in the equations, we can drop the primes altogether and write in the following most general form the equations of motion of a material particle moving with respect to a system of coordinates with origin on the surface of the sun and rotating with the sun :

$$\left.\begin{aligned}
&\text{Equatorwards} \\
&m\frac{d^2x}{dt^2} = X + 2m\omega v\sin\phi + m\omega^2\sin\phi(R\cos\phi + x\sin\phi + z\cos\phi) \\
&\text{Westwards} \\
&m\frac{d^2y}{dt^2} = Y - 2m\omega(u\sin\phi + w\cos\phi) + m\omega^2 y, \\
&\text{Upwards} \\
&m\frac{d^2z}{dt^2} = Z + 2m\omega v\cos\phi + m\omega^2\cos\phi(R\cos\phi + x\sin\phi + z\cos\phi).
\end{aligned}\right\} \quad \dots \quad (10)$$

If we restrict ourselves to small regions of the sun's surface as we shall when considering dark markings and prominences, the terms containing x, y and z can be neglected in comparison with those containing R and the equations (10) can be simplified as follows :

$$\left.\begin{aligned}
&\text{Equatorwards} \\
&m\frac{d^2x}{dt^2} = X + 2m\omega v\sin\phi + m\omega^2 R\sin\phi\cos\phi \\
&\text{Westwards} \\
&m\frac{d^2y}{dt^2} = Y - 2m\omega(u\sin\phi + w\cos\phi) \\
&\text{Upwards} \\
&m\frac{d^2z}{dt^2} = Z + 2m\omega v\cos\phi + m\omega^2 R\cos^2\phi
\end{aligned}\right\} \quad \dots \quad (11)$$

These equations show that in addition to the forces X, Y, and Z which are responsible for the initial motion of the particle there appear other forces which modify the motion. These additional forces are clearly apparent or subjective forces, for they become zero if the particle is at rest or if the angular velocity of the sun's rotation becomes nil. The first of the equations (11) shows that the particle has a southward acceleration $2\omega v \sin\phi$ proportional to its westward component of velocity and to the angular velocity of rotation of the sun. This means that if the particle tries to move from east to west it is affected by an apparent force which deflects its path towards some point south of west. The southward acceleration also contains a term $\omega^2 R \sin\phi\cos\phi$ which for a given latitude ϕ depends only on the angular velocity of rotation of the sun and therefore acts upon the particle even when it is not impelled to move by the force X. This term is clearly the equatorward component of the centrifugal force due to the sun's rotation and it must exist so long as we can regard the sun as a sphere without any flattening at the poles like the earth. It may be noted that all measurements of the solar disc have failed to reveal any departure from the perfectly circular shape. We shall not consider the possible reasons for this perfect symmetry of the sun's shape, but accept it as an observed fact. An uncompensated equatorward force $\omega^2 R \sin\phi\cos\phi$ must therefore act on a particle on the sun. If the particle is embedded in the dense layers of the sun's atmosphere the effect of this force may be observable only as a very slow equatorward drift of the particle. The slow equatorward movement of sunspots and of the prominences of the sunspot belt may be due to the effect of this force. If the particle happens to be in the thinner regions of the solar atmosphere, for example in the coronal atmosphere, this force may be expected to produce a considerable equatorward velocity of the particle.

The second of equations (11) shows that the westward acceleration $\frac{d^2y}{dt^2}$ contains two terms $-2\omega u \sin\phi$ and $-2\omega w \cos\phi$. The first term means that the particle has an eastward acceleration due to a deflective force proportional to the angular velocity of the sun's rotation and to the southward component of its velocity. Hence if the particle tries to move along a meridian from higher to lower latitudes, its path is deflected *towards the east* and therefore makes an angle with the meridian. The second term indicates that the particle is acted upon by a deflective force proportional to the vertical component of its velocity and to the angular velocity of the sun's rotation or, in other words, if the particle tries to move vertically upwards its path is deflected *towards the east* and therefore makes an angle with the vertical. From the third of equations (11) we see that the particle is acted upon by a deflective force $2\omega v \cos\phi$ which deflects its horizontal east-west trajectory upwards or away from the surface of the sun. The vertical acceleration $\frac{d^2z}{dt^2}$ also contains a term $\omega^2 R \cos^2\phi$, which is clearly the vertical component of the centrifugal force due to the sun's rotation. This term must be responsible for causing a variation in the actual gravity on the sun from the equator to the poles. But this term will play no part in the considerations set forth in this paper, since we shall assume according to the usual practice that in prominences gravity is compensated by radiation pressure under normal conditions. The above equation of motion also shows that a particle moving from west to east is forced downwards by an apparent deflective force $-2\omega v \cos\phi$ and conversely an east-to-west trajectory is deflected upwards by an apparent force $2\omega v \cos\phi$. This may be responsible for the curving back of the tops of some prominences to the surface of the sun and for the occurrence of spiral motion observed in some cases. The force $2\omega v \cos\phi$ clearly sets a limit to the length of a marking. We can now examine the effect of the equations of motion (11) on a mass of gas ejected from some layer of the solar envelope or from the photosphere. The process responsible for the ejection of the gas does not matter for the purposes of the present paper, but we may perhaps regard it as a sudden and temporary increase of radiation in a limited region of the layer concerned. Since under normal circumstances the effect of gravity is compensated by radiation pressure this sudden increase in radiation will lead to the ejection of a mass of gas in the form of a jet. The velocity with which the jet will travel outwards will obviously depend upon the actual increase in radiation and upon the friction between the rising mass of gas and the surrounding atmosphere. It seems reasonable to expect that this increase of radiation has an average value and therefore the rising mass of gas will emerge into the coronal atmosphere with a sort of normal velocity determined by the viscosity of the solar material which it has traversed during the travel. Now, experimental measurements of the angular velocity of the sun's rotation at the photosphere and in different levels of envelope have shown very little variation of angular velocity with elevation, so that we may take ω to have a practically constant value over the whole region from the photosphere to the corona. Therefore a mass of gas emerging from any part of this region will have the same initial angular velocity ω as the sun's surface and therefore its motion will be determined by the equations (11). Accordingly it will be deflected from the vertical towards the east and its equatorward motion under the action of the propelling force $\omega^2 R \sin\phi . \cos\phi$ will follow a trajectory inclined to the meridian towards the east. This conclusion is, however, directly opposed to observational data according to which the top of a dark marking is inclined towards the west with respect to the vertical and also the inclination of a dark marking to the meridians is such that its equatorial end lies to the west of the polar end. Now, Perepelkin[1] has concluded that the quiescent prominences form in the low layers of the sun. It seems to us very likely that the eruption which supplies the material for the formation of a prominence has its seat in the interior of the sun below the photosphere. We therefore assume that the mass of gas which gives rise to an ordinary dark marking originates in the interior of the sun where the angular velocity is very high and that on reaching the coronal region it still has an angular velocity ω' much higher than the angular velocity ω of the sun's surface. When a mass of gas emerges from the interior with the angular velocity ω' it will have, relatively to the sun's surface, an angular velocity $\omega'-\omega$ in the same direction in which the sun's surface, is observed to rotate. The dynamical situation will therefore be equivalent to the motion of a particle with respect to a system of coordinates rotating with an angular velocity $-(\omega'-\omega)$. The appropriate dynamical equations are obtained directly from (11) by making the necessary substitution and we have

Equatorwards

$$m\frac{d^2x}{dt^2}=X-2m(\omega'-\omega)v\sin\phi+m(\omega'-\omega)^2R\sin\phi\cdot\cos\phi, \quad (a)$$

Westwards

$$m\frac{d^2y}{dt^2}=Y+2m\,(\omega'-\omega)\,(u\sin\phi+w\cos\phi), \quad (b) \qquad \ldots \quad (12)$$

Upwards

$$m\frac{d^2y}{dt^2}=Z-2m\,(\omega'-\omega)\,v\cos\phi+m\,(\omega'-\omega)\,R\cos^2\phi. \quad (c)$$

From these equations it is evident that the deflection of the uprising jet of gas from the vertical and from the meridians is in agreement with observational data so far as the direction of inclination is concerned. We now proceed to examine how far the inclination derived from equations (12) agree quantitatively with observed values.

The jet of gas may be expected to travel through the coronal atmosphere with a steady velocity horizontally and perhaps also in the upward direction ; in fact, if the jet is to make a definite angle with the vertical and with the meridians, it must move with a steady velocity. We note that the constancy of speed implies that all the forces acting on the moving mass of gas mutually

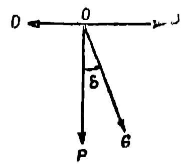

FIGURE 3

balance each other. Let G be the force (per unit mass) which is responsible for the equatorward motion of the gas acting in the direction OG (see Fig. 3) and let OP represent the direction in which the jet actually travels so that $\widehat{GOP}$ is the angle we want to determine. We assume that the curvature of the trajectory is negligible, that is, the jet moves practically in a straight line. Now the deflective force $2(\omega'-\omega).u \sin\phi$, u being the equatorward velocity, must act at right angles to the direction OP, that is, in the direction OD. We resolve the force G into two components in the directions OP and OB. Then in the steady state the component along OB must be balanced by the deflective force along OD and the component along OP must be balanced by the force of frictional resistance which, as usual, is given by $P=f.c$, where c is the velocity of the moving body and f is the coefficient of friction. Accordingly we have the following relation on putting $c=u$

$$G.\sin\delta=2(\omega'-\omega)\,u\sin\phi \qquad [\textit{vide} \text{ eqn. (12) (b).}]$$
$$G.\cos\delta=f.u,$$

whence we obtain

$$\tan\delta=\frac{2(\omega'-\omega)\sin\phi}{f}. \qquad \dots \quad (13)$$

In order to evaluate δ numerically from (13) we must know the numerical value of the frictional constant. We can estimate the magnitude of the frictional constant in the following way. We assume the moving mass of gas to be a sphere of radius r and its velocity u. Then, according to the well-known law of Stokes the gas sphere will experience a total resistance $\overline{P}=6\pi\eta_k ru$, where η_k is the coefficient of viscosity of coronal matter, while moving through the coronal atmosphere. Therefore

$$f=6\pi\eta_k r/\tfrac{4}{3}\pi r^3\rho_p=\frac{9\eta_k}{2r^2\rho_p}. \qquad \dots \quad (14)$$

Now η_k can be estimated with the help of Jeans's theory,[3] according to which the viscosity of stellar matter is given by $\eta=\frac{4}{15}.\frac{aT^4}{k\rho c}$, where $k=\frac{c\rho}{\mu'T^{3\cdot5}}\left(1+\frac{h\nu}{RT}\right)$ as given by Kramers's theory of electron capture. The temperature of coronal matter (in the region where prominences appear) may be taken to be about 10^{-4} times the central temperature of the sun. The density of coronal matter (ρ_k) may be taken as about 10^{-3} times the density of prominence matter ρ_p. Now a prominence, which is composed almost entirely of atomic hydrogen with a very slight admixture of metals and rare gases, contains about 10^{13} atoms of hydrogen per cubic centimetre; therefore, $\rho_p=1\cdot7\times10^{-11}$ gm./cm.^{3}, taking the mass of a hydrogen atom to be $1\cdot7\times10^{-24}$ gm. Hence we have $\rho_k=1\cdot7\times10^{-14}$ gm./cm.^{3} which is about 3×10^{-16} times the central density of the sun. Using these values of temperature and density in Jeans's formula we obtain $\eta_k=10$ times the viscosity at the centre of the sun. The same formula gives $\eta=12\cdot2$ at the centre of the sun. Therefore, $\eta_k=122$. Substituting this value of η_k in (14) we have

$$f=\frac{9\times122}{2\times10^{18}\times1\cdot7\times10^{-11}}$$
$$=3\cdot2\times10^{-5}, \qquad \dots \quad (15)$$

putting $r = 10^9$ cm. The reason for putting $r = 10^9$ cm. is that the radius of the mass of gas ejected should be approximately the same as that of an average area of the sun's surface affected by an eruption and that the latter quantity appears to be of the order of 10^9 cm. In order to evaluate δ numerically we must know the numerical value of ω' which is the only unknown left in equation (13), ω being known from observation. In a well-known paper Jeans[3] has shown that the flow of radiation outwards from the interior of a star behaves like a convection current of matter and in fact the viscosity arising from the flow of radiation is more important than material viscosity. It therefore follows that there must be a steady decrease in the angular velocity of rotation from the centre of a star to its surface, but there will be a tendency for the interior of the star to attain a uniform angular velocity of rotation ω^* which must satisfy the relation† $\varpi^2\omega^{2*} = \text{const}$. As most of the mass of a star is contained in a sphere of radius barely $\frac{1}{3}$ the radius of the star the average velocity of rotation should be about 10 times the velocity of rotation observed on the surface. In the case of the sun[4] the average period of rotation should therefore be from 2 to 3 days. It is to be noted however that the relation $\varpi^2\omega^* = \text{const.}$ requires that ω^* should increase from the equator to the poles which is contrary to what is observed on the surface of the sun. Nevertheless this contradiction between theory and observation does not mean that the interior of the sun does not rotate in the way required by theory. The decrease of angular velocity from the equator to the poles, the so-called polar retardation, observed on the surface of the sun may be due to the superposition of a subsidiary mechanism. In fact such a mechanism has been indicated by Eddington.[5] According to Eddington the sun does not satisfy *v.* Zeipel's theorem. A departure from *v.* Zeipel's theorem causes the temperature to rise at the equator and fall at the pole or conversely, so that a pressure gradient is established along the meridians and circulatory currents are initiated in the meridian planes. These primary meridional currents must, however, be deflected to the east or west and tend to become parallel to the parallels of latitude with increasing latitude. These circulatory currents are, according to Eddington, responsible for the polar retardation of rotational velocity as observed on the surface of the sun. It seems likely that these currents would be confined to the upper levels and would not extend much below the photosphere. The interior of the sun will rotate according to the law $\overline{\varpi}^2\omega^* = \text{const}$. We assume that the gas, which ultimately forms into prominences and dark markings, originates in some level below the level of the circulatory currents. At this level we shall have

$$\varpi^2\omega^* = \varpi_0^{\,2}\omega_0^*$$

or,

$$R^{*2}\cos^2\phi . \omega^* = R^{*2}\omega_0^*$$

† ϖ is the distance from the axis of rotation.

or,
$$\omega^* = \omega_0^* \sec^2 \phi \tag{16}$$

where ω^* is the angular velocity at latitude ϕ, ω_0^* is the angular velocity at the equator and R^* is the radius of the sun at the level concerned. Now, ω' is sure to be smaller than ω^*. Let us put $\omega' = \frac{1}{n} . \omega^*$, where n is a number greater than 1. Then, using this relation and substituting (16) in equation (13), we have

$$\tan \delta = \frac{2}{f} \left(\frac{1}{n} \omega_0^* \sec^2 \phi - \omega \right) \sin \phi$$

or
$$\tan \delta = \frac{2}{f} \left(\frac{1}{n} \omega_0^* \tan \phi . \sec \phi - \omega \sin \phi \right). \quad \ldots \quad (17)$$

Since the average angular velocity of the interior of the sun is about 10 times the angular velocity of rotation observed on the surface of the sun we may put on the average $\frac{1}{n} \omega_0^* = 10 \omega_0$. Then using the value of f from (15) and the values of ω at different latitudes from d' Azambuja's empirical formula we can evaluate δ for different latitudes from equation (17). The results of these computations are set forth in column 4 of the following table.

TABLE

Latitude (ϕ)	$\omega \times 10^5$ sidereal	δ (observed)	δ (theoretical)
0°	0.292	4°	0°
5°	0,291	11°	8°
10°	0.291	17°	16.°3
15°	0.290	28°	24°·7
20°	0.288	44°	32°·7
25°	0.286	45°	40°·7
30°	0.283	58°	48°
35°	0.281	61°	55°·3
40°	0·278	73°	61°·8
45°	0·276	70°	67°·5
50°	0·273	67°	72°·8
55°	0·270	73°	77°·2
60°	0·267		80°

For comparison with observation the values of δ derived by Royds and Salaruddin [6] from the measurement of dark markings as recorded in the Meudon charts for a complete 11-year cycle from 1923 to 1933 are given in column 3. Column 2 of the table gives the values of $\omega \times 10^5$ for different latitudes as computed from d'Azambuja's formula. It is clear from the table that the theoretically calculated values of the inclination of dark markings to the meridians agree exceedingly well with those derived from observation. This close agreement between theory and observation shows up very strikingly in

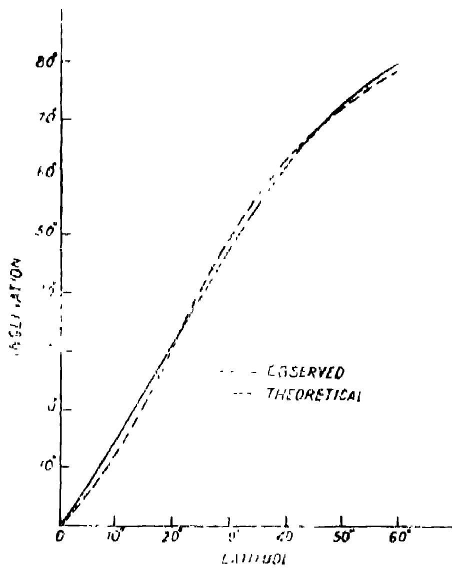

FIGURE 4

Fig. 4. It is evident therefore that the mechanism considered in this paper is perfectly capable of explaining qualitatively as well as quantitatively the westward inclination of dark markings to the meridians and its variation with latitude, which are two of the most striking features of dark markings and which have so far received no satisfactory explanation. Another puzzling feature, namely that a suddenly appearing dark marking has, almost at its first appearance, its direction inclined to the meridian at an angle appropriate to the latitude of its occurrence, is an obvious consequence of the present theory.

We have already seen that, according to the mechanism here considered, the height of a prominence or dark marking should be inclined to the west of the vertical which is in qualitative agreement with the observation of Mlle Roumens [7] of the Meudon observatory. Mlle Roumens obtained an average tilt of 10°W which is the average value derived from measurements of 171 dark markings in low and middle latitudes. Apparently she finds various inclinations, but she mentions that the inclinations show a peak at 7° and another more pronounced one at 22°, that is to say, the value of the inclination to the vertical is more often 22° than 7°. For comparison with these observations we can calculate the angle δ which the height of a dark marking should make with the vertical in the same way as for the meridional inclination. δ will again be given by the formula (13) in which $\sin\psi$ has now to be replaced by $\cos\psi$ [*vide* eq. (12) (*d*)]. The value of δ thus calculated comes out to be of the order of 50° to 60° which is much larger than the observed value. The reason for this discrepancy is probably that the assumption of steady motion which lies at the basis of the theoretical derivation is not justified in the case of the upward motion. This is perhaps also what is to be expected. The frequent occurrence of streamers in prominences is probably to be looked upon as a result of the unsteadiness of the upward motion. Arguments, both theoretical and experimental, in favour of this conclusion have been given by H. Zanstra in a very recent paper.[8] If the outward radial motion is unstable, there can be no steady inclination between the height of a dark marking and the vertical; but there will still be a rough average inclination for a given time during which the upward motion has existed. Let a jet of gas emerge out of the chromosphere at the equator with a velocity 2V and rise to the height of a typical prominence, namely about 50000 km. At that height its upward velocity is nil, so that the average upward velocity of the jet during the ascent is V. Then from the second of the equation (12) we have

$$\frac{d^2y}{dt^2}=2(\omega_0{}'-\omega_0).V=18\,\omega_0 V.$$

Integrating this equation we get

$$y=9\omega_0 Vt^2. \qquad \dots \quad (18)$$

Now, according to Pettit the most frequent radial velocities measured in quiescent prominences range from 5 km./sec. to 10 km./sec. and velocities higher than 15 km./sec. are infrequent. We may therefore take 5 km./sec. to be the average valocity with which the jet of gas responsible for the formation of a dark marking travels through the coronal atmosphere. Then $t=50000/5$ secs. $=10^4$ secs. Using these values of V and t and the value of ω_0 from the table above we get from (18) the westward displacement of the top of an average dark marking :

$$y=13150 \text{ km}.$$

Then the inclination to the vertical is given by

$$\tan\delta=\frac{13150}{50000}=0{\cdot}263$$

or

$$\delta=14^{\circ}{\cdot}75.$$

This rough estimate of the average inclination of a dark marking or prominence is in quite satisfactory agreement with the value derived by Mlle Roumens from observation.

It may be noted in this connection that the mechanism here considered also gives an indication of the magnitude of the breadth of a dark marking. If the whole of the jet above the chromosphere is effective in absorbing photospheric radiation then obviously the breadth of the dark marking should roughly be the westward displacement of the top from the vertical *plus* the diameter of the jet. At the equator the westward displacement for a typical prominence is of the order of 13000 km., so that the breadth of the corresponding dark marking should be of the order of 33000 km. taking the radius of the jet to be about 10^9 cm. or 10000 km. The ratio of height to breadth at the equator will therefore be approximately 1·5 : 1. But this ratio cannot be the same at all latitudes, since the westward inclination to the vertical increases from the equator towards the higher latitudes. In fact, the westward displacement at any latitude ϕ is given by

$$(\omega_0' \sec \phi - \omega \cos \phi)\ Vt^2.$$

Consequently the breadth of a dark marking should increase as we proceed from the equator towards the higher latitudes so that the ratio of the height to breadth would decrease with increasing latitude. But beyond a certain latitude where the length of a dark marking is practically parallel to the parallels of latitude the breadth will be about the same as the diameter of the jet, since the westward tilt of the dark marking and the length of the dark marking will practically lie in a vertical plane normal to the meridians. At high latitudes therefore, the breadth of a typical dark marking would be of the order of 20000 km., which is in fair agreement with observation, and the ratio of the height to breadth would be of the order of 2·5 : 1. Now the equatorward force $(\omega'-\omega)^2$ R $\sin\phi \cos\phi$ is zero at the equator and increases with the latitude; consequently the length (by which we mean the extension across the parallels of latitude) of the marking will be equal to the diameter of the jet or about 20000 km., so that its breadth (by which we mean its extension across the meridians) will be greater than its length. This means that at the equator the marking will appear to be parallel to the equator, and slightly above the equator the marking will tend to be as long as it is broad. From a consideration of the variation of the force $2(\omega'-\omega)v \cos\phi$ [*vide* eq.(12) (*c*)] along with the variation of the force $(\omega'-\omega)^2$ R $\sin\phi\cos\phi$, it follows also that the length of a dark marking should increase as we proceed from the equator towards higher latitudes. In general, at least at the middle and high latitudes, the length of a dark marking can be expected to be much greater than its breadth, thus accounting for the predominantly linear appearance of dark markings. It is difficult to estimate the order of magnitude to be expected for the ratio of the length to breadth of a dark marking formed by a single jet, but it seems likely that even at high latitudes it will not exceed 5 : 1 or 6 : 1. The extraordinarily long markings of high latitudes appear however to be produced by a numbar of jets arranged along a parallel of latitude ; in such cases the ratio of the length to breadth would be several times more. It is a curious fact that the several jets should arrange themselves along a parallel of latitude at high latitudes the reason for which is not clear; but it seems possible that jets have a tendency to form along the paths of the internal circulating currents which of course must be parallel to the parallels of latitude at high latitudes and become more and more parallel to the meridians as we approach the equator practically in the same way as the directions of dark markings do. This may be the reason why several jets form along the length of a long dark marking; if that be so, the occasional formation of extraordinarily long markings becomes understandable.

In conclusion, I wish to emphasise that the object of the present investigation has been rather to evolve a self-consistent hypothesis capable of accounting for some of the major peculiarities of solar dark markings than to develop a complete theory. My thanks are due to Dr. C. W. B Normand, C.I.E., Director-General of Observatories in India, a discussion with whom about the peculiarities of dark markings led me to undertake this investigation.

ADDENDUM

This paper was written in June, 1939; but its publication has so far been postponed as certain points arising out of the mechanism suggested herein needed consideration. In the mean time Mr. P. R. Chidambara Iyer of this observatory has rightly pointed out to me in a discussion in a different connection that he has held for a long time that a dark marking having an inclination to the meridians attains its minimum area only when its direction is radial to the disc and not when it is at the central meridian. This certainly throws doubt on the validity of Mlle Roumens' conclusion that prominences have a predominantly westward tilt to the vertical. This doubt has been clearly raised by Waldmeier in a very recent paper (*Astro. Mitteilungen, Zürich*, 1939) which has just come to my notice. According to Waldmeier the fact that a dark marking has a minimum area at an eastern longitude and not at the central meridian is a necessary consequence of the inclination of the dark marking to the meridians. Against this one may point out that Mr. M. Salaruddin has found (*K. O. B.*, Vol. IV, No. 96, p. 297) that dark markings without any inclination to the meridians show a preponderance of western areas over eastern areas. This dissymmetry points to the existence of a west tilt. It seems probable that the observed dissymmetry in the case of the markings inclined to the meridians is due jointly to the causes envisaged by Roumens and by Waldmeier. This point is under examination. Should the existence of the west tilt be definitely established, it would find a plausible theoretical cause in the mechanism proposed in the present paper.

SOLAR PHYSICS OBSERVATORY,
KODAIKANAL.
December, 1939.

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